International Journal of Theoretical Physics, Vol. 25, No. 10, 1986

Hadron Fields from Anisotropic Space and Their Interaction

S. S. De[1]

Received June 3, 1986

Hadron fields are constructed from constituent fields in the anisotropic microdomain, regarded as Finslerian, which were discussed in an earlier paper. The general one-particle hadron states are formulated. The many-particle hadron states are also formed as direct products of one-particle states. Then the field theory of hadrons in the macrodomain is discussed and formal calculations are made for the reaction amplitude of the meson–baryon interaction and compared with that of our previous model. It is found that the amplitude is dependent on that of the $\pi\pi$ interaction together with a factor arising from the "rearrangement" of the constituents. This factor provides an extra momentum dependence that leads to the "energy-dependent coupling" which makes it possible to apply perturbation technique in strong interactions, as discussed in our earlier papers.

1. INTRODUCTION

In a recent paper (De, 1985), physical fields in microspace, regarded as Finslerian, are constructed. There, the "handedness" necessary in the structure of the subnuclear particles for the space–time formulation of the internal symmetry is connected with the extension of the particle in the microdomain. The field equations for these physical fields as well as the corresponding equations for the macrodomain, the Minkowskian space, are derived with the introduction of a kind of "averaging."

It is proposed there that the physical field function $\psi(x, \nu)$ is dependent on the line support element (x, ν) of the Finsler space with the metric $g_{ij}(x, \nu)$ and the connection coefficients $p^{\mu}_{hj}(x, \nu)$ depending not only on the position coordinates $x=(x^0, x^1, x^2, x^3)$, but also on some directional variable $\nu=(\nu^0, \nu^1, \nu^2, \nu^3)$. In this model the extension of the subnuclear particle is of composite type and the constituents are situated at neighboring points

[1]Department of Applied Mathematics, University College of Science, Calcutta University, Calcutta 700009, India.

of the microdomain, less than 10^{-13} cm apart. In fact, it is proposed there that the constituents lie on some spacelike surface in the space of coordinates x^μ ($\mu = 0, 1, 2, 3$), the underlying manifold, which is essentially Minkowskian. Furthermore, the extension in the microdomain is such that the constituents must be on a geodesic in that space (the Finsler space), and thus "every unit field orthogonal to this geodesic is parallel along it" (Chowdhuri, 1981). In fact, this curve is the autoparallel curve whose tangent vectors result from each other by successive infinitesimal parallel displacement of the type [using the notations of Rund (1959)]

$$dv^i = -p^i_{hj}(x, v) v^h \, dx^j \tag{1}$$

The neighboring points on the autoparallel curve where the constituent particles lie (or the corresponding fields depend) are x^μ, $x^\mu + dx^\mu$, $x^\mu - dx^\mu, \ldots$, and the (distance) element dx^μ is quantized by setting

$$dx^\mu = i\varepsilon\hbar\gamma^\mu \tag{2}$$

with ε a real, positive parameter, which may vary from 0 to l, the fundamental length, of the order of 10^{-13} cm, and γ^μ ($\mu = 0, 1, 2, 3$) the Dirac matrices.

From an assumed fundamental property of the field functions of the constituents or particles, which is the physical equivalence of the field functions (or wave functions) at neighboring points on the autoparallel curve, it is possible to deduce an equation for the microfield of the form

$$i\hbar\gamma^\mu(\partial_\mu - \hat{G}^l_\mu \partial'_l)\psi(x, v) = 0 \tag{3}$$

with

$$p^\mu_{hj}(x, v) v^h = \hat{G}^\mu_j \tag{4}$$

Also, it is shown that an "averaging" over the microdomain can yield the usual Dirac equation for the field in the macrodomain.

In the present paper this formalism is extended for the construction of hadron fields from the constituent fields in Finsler space. Also, the interactions among these hadron fields in the macrodomain are formulated. In Section 2, the particle space formed of the particle states is constructed, and in Section 3, the interactions of hadrons are discussed. Finally, in Section 4, the amplitudes for these reactions are formally calculated and compared with the amplitude of our previous model (Bandyopadhyay and De, 1975a; De, 1983), which satisfies the requirements of the S-matrix theory, such as analyticity, unitarity, and crossing. Also, the field theory is applicable there because the "energy-dependent coupling" leads to the perturbation technique being applicable to the strong interactions. Thus, the basis of the previous model is deepened, in addition to the establishment of the theory of the basic structure of hadrons.

2. THE PARTICLE SPACE

We now construct the field functions (or wave functions) of the hadrons from those of the constituents, which are spinors (or may be Majorana spinors). As has already been stated, the fields of these constituents are functions of the line support elements in the Finsler space, the microdomain. We first consider the particle states only for mesons, the subset of the whole hadron states. Of course, the generalization to the whole hadron state will be straightforward. The pions (pseudoscalar and vector) are formed by the two constituents from the leptons μ^+, μ^-, ν_μ (Bandyopadhyay, 1984) and thus the wave (or field) functions of the pions must be functions of the two neighboring line support elements on the autoparallel curve of the microspace, their coordinates x^μ being on the spacelike surface of the underlying manifold (the Minkowskian x^μ space). The quantized distance dx^μ given by (2) satisfies

$$dx^\mu \, dx_\mu = (dx)^2 = i^2 \varepsilon^2 \hbar^2 \gamma^\mu \gamma_\mu = -4\varepsilon^2 \hbar^2 < 0$$

Now we form the one-particle pion state as follows:

$$|\psi(p)) = \int e^{ipx} \, d^4x \, d^4\nu \, d\varepsilon$$
$$\times \Phi_{AB}(x, \xi_1, \xi_2) \bar{\psi}^{(-)A}(x, \xi_1) \psi^{(-)B}(x, \xi_2)|0) \tag{5}$$

where $\bar{\psi}^{(-)A}$ and $\bar{\psi}^{(-)B}$ are, respectively, the creation operators for the particle (the lepton) and its charge conjugate, the creation operator for the antiparticle. The destruction operators for the particle and antiparticle are, respectively, $\psi^{(+)A}$ and $\bar{\psi}^{(+)B}$. The sub- and superscripts A and B are (α, a) and (β, b), respectively, where α, β correspond to spin (Dirac) indices of the lepton constituents and a, b the orbital angular momenta of the constituents (Bandyopadhyay, 1984). The field operators $\bar{\psi}^{(-)A}$ and $\psi^{(-)B}$ are functions of the line support elements $(x^\mu - \frac{1}{2}dx^\mu, \nu^\mu - \frac{1}{2}d\nu^\mu)$ and $(x^\mu + \frac{1}{2}dx^\mu, \nu^\mu + \frac{1}{2}d\nu^\mu)$, respectively. In expression (5) the following notations have been used:

$$\begin{aligned}
&\bar{\psi}^{(-)A}(x^\mu - \tfrac{1}{2}dx^\mu, \nu^\mu - \tfrac{1}{2}d\nu^\mu) \\
&\quad = \bar{\psi}^{(-)A}(x^\mu - \tfrac{1}{2}i\varepsilon\hbar\gamma^\mu, \gamma^\mu - \tfrac{1}{2}d\nu^\mu) \\
&\quad \equiv \bar{\psi}^{(-)A}(x^\mu, -\varepsilon, \nu^\mu - \tfrac{1}{2}d\nu^\mu) \\
&\quad = \bar{\psi}^{(-)A}(x^\mu, \xi_1^\mu)
\end{aligned} \tag{6}$$

$$\begin{aligned}
&\psi^{(-)B}(x^\mu + \tfrac{1}{2}dx^\mu, \nu^\mu + \tfrac{1}{2}d\nu^\mu) \\
&\quad = \psi^{(-)B}(x^\mu + \tfrac{1}{2}i\varepsilon\hbar\gamma^\mu, \nu^\mu + \tfrac{1}{2}d\nu^\mu) \\
&\quad = \psi^{(-)B}(x^\mu, \xi_2^\mu)
\end{aligned} \tag{7}$$

with

$$\xi_1^\mu = (-\varepsilon,\ \nu^\mu - \tfrac{1}{2} d\nu^\mu)$$
$$\xi_2^\mu = (+\varepsilon,\ \nu^\mu + \tfrac{1}{2} d\nu^\mu) \tag{8}$$
$$\xi^\mu = (0,\ \nu^\mu)$$

Note that the normalization factor in (5) has been absorbed into the "inner" amplitude function $\Phi_{AB}(x, \xi_1, \xi_2)$ that has been introduced for construction of the one-particle (pion) state from its constituents. For the general one-particle hadron state we can easily generalize expression (5) by summing over the one-particle states of different hadrons; that is,

$$\begin{aligned} |\psi(p)) = \sum_i \int & e^{ipx}\, d^4x\, d^4\nu\, d\varepsilon \\ & \times \Phi(x, \xi_1, \xi_2, \ldots \xi_{k+n}) A_1^i A_2^i \cdots A_k^i B_{k+1}^i \cdots B_{k+n}^i \\ & \times \bar{\psi}^{(-)A_1^i}(x, \xi_1) \cdots \bar{\psi}^{(-)A_k^i}(x, \xi_k) \\ & \times \psi^{(-)B_{k+1}^i}(x, \xi_{k+1}) \cdots \psi^{(-)B_{k+n}^i}(x, \xi_{k+n}) |0) \end{aligned} \tag{9}$$

Here, i, which may be an abstract index or can indicate the quantum numbers, characterizes the hadron; k and n indicate the numbers of the constituent particles or antiparticles, respectively, and are dependent on the particular hadron or the index i.

Now the one-particle state (5) can be written as

$$|\psi(p)) = \int e^{ipx}\, d^4x\, d^4\nu\, d\varepsilon\, \tilde{\Phi}_{AB}(x, \varepsilon, \nu) \bar{\psi}^{(-)A}(x, \nu) \psi^{(-)B}(x, \nu)|0) \tag{10}$$

where we have used equations (3) and (4) satisfied by the operators $\bar{\psi}^{(-)A}$ and $\psi^{(-)B}$ and

$$\begin{aligned} & \Phi_{AB}(x, \xi_1, \xi_2) \\ & \quad \equiv \Phi_{AB}(x^\mu, \varepsilon, \nu^\mu - \tfrac{1}{2} d\nu^\mu, \nu^\mu + \tfrac{1}{2} d\nu^\mu) \\ & \quad = \Phi_{AB}\left(x^\mu, \varepsilon, \nu^\mu + \frac{i\varepsilon\hbar}{2} p_{hj}^\mu(x, \nu) \nu^h \gamma^j, \nu^\mu - \frac{i\varepsilon\hbar}{2} p_{hj}^\mu(x, \nu) \nu^h \gamma^j \right) \\ & \quad \equiv \tilde{\Phi}_{AB}(x, \varepsilon, \nu) \end{aligned} \tag{11}$$

Writing $\mathbf{l}_1$, $\mathbf{s}_1$ and $\mathbf{l}_2$, $\mathbf{s}_2$ for the orbital and spin angular momenta for the two constituents of the pion, one of which, say the first being the "central" particle (Bandyopadhyay, 1984; Bandyopadhyay and De, 1975a), the total angular momentum of the second $\mathbf{j}_2 = \mathbf{l}_2 + \mathbf{s}_2$ gives the isospin of the pion. The total angular momentum $\mathbf{J} = \mathbf{j}_1 + \mathbf{j}_2$, where $\mathbf{j}_1 = \mathbf{l}_1 + \mathbf{s}_1$, becomes the spin of the hadron (pion, in this case). As for the example, for π^- and ρ^-, $\beta = 2$,

$b=2$ (it is in the space of preferred direction ν, the Finsler space), $j_2=|\mathbf{j}_2|=1$ and $(j_2)_3=-1$ give rise the correct isospin state of the particles. Also, we can have the correct spins of the pseudoscalar and vector mesons from the total angular momenta of their constituents in this model (Bandyopadhyay, 1984). The relevance of the preferred direction of the microdomain is in the orbital angular momenta of the constituent particles. In fact, in this model, the constituents, which are the leptons and the mesonic system composed of a muon-antimuon pair, move in a harmonic oscillator potential with orbital angular momentum $\frac{1}{2}\hbar$ in such a way that the two values of the third component of the orbital angular momentum represent the two states of matter: the particle and antiparticle. Such half-orbital angular momentum can be acceptable in this anisotropic space, the space of preferred direction, and is relevant for determination of the internal symmetry of hadrons. In fact, in this theory, as pointed out earlier, the internal quantum numbers, such as isospin, strangeness, and baryon number, are closely related to the internal angular momenta of the constituents of the hadrons. Thus the manifestations of the direction variable (of the microdomain) into the macrodomain are in these quantum numbers, such as baryon number, strangeness, etc., and are carried by the orbital angular momenta of the constituents.

As in the previous paper (De, 1985), we decompose the field functions into x parts and ν parts, where it has been shown that the x parts satisfy the Dirac equation. Thus we write

$$\begin{aligned}\bar{\psi}^{(-)A}(x,\nu)&=\bar{\psi}^{(-)\alpha}(x)\bar{\psi}^{a}(\nu)\\ \psi^{(-)B}(x,\nu)&=\psi^{(-)\beta}(x)\psi^{b}(\nu)\end{aligned}\tag{12}$$

Then we have from (10),

$$\begin{aligned}|\psi(p))&=\int e^{ipx}\,d^4x\,d^4\nu\,d\varepsilon\\ &\quad\times\bar{\psi}^{(-)\alpha}(x)\psi^{(-)\beta}(x)\bar{\psi}^{a}(\nu)\psi^{b}(\nu)\tilde{\Phi}_{AB}(x,\varepsilon,\nu)|0)\\ &=\int d^4x\,e^{ipx}\bar{\psi}^{(-)\alpha}(x)\psi^{(-)\beta}(x)G^{ab}_{AB}(x,l)|0)\end{aligned}\tag{13}$$

where

$$G^{ab}_{AB}(x,l)=\int_0^l d\varepsilon\int d^4\nu\,\bar{\psi}^{a}(\nu)\psi^{b}(\nu)\tilde{\Phi}_{AB}(x,\varepsilon,\nu)\tag{14}$$

Now it is to be noted that when the integrand in (14) is being "averaged," according to the integration therein, from the internal space to the macrospace, the Minkowskian space, the preferred direction is "lost" and as such there is no specific $(j_1)_3$ and $(j_2)_3$ values or consequently no specific j_3 value of the concerned hadron. However, as has already been mentioned, the direction variable of the internal space manifests itself onto the macrospace through the quantum numbers such as baryon number, hypercharge, and isospin given by the sub- and superscripts of the G-function of (14). Also, the moduli of $\mathbf{j}_1$, $\mathbf{j}_2$ and that of $\mathbf{j}$ are maintained in the averaged space (the macrospace), where $|\mathbf{j}_1+\mathbf{j}_2|=|\mathbf{j}|=0$ for π-mesons and $|\mathbf{j}|=1$ for ρ-mesons.

For a general one-particle hadron state, we can generalize expression (13) [or can obtain from (9)] as follows:

$$|\psi(p)) = \sum_i \int d^4x\, e^{ipx} \bar{\psi}^{(-)\alpha_1^i}(x) \cdots \bar{\psi}^{(-)\alpha_k^i}(x) \times \psi^{(-)\beta_{k+1}^i}(x) \cdots \psi^{(-)\beta_{k+n}^i}(x) G_{D^i}^{l^i}(x, l)|0) \tag{15}$$

with

$$G_{D^i}^{l^i}(x, l) = \int_0^l d\varepsilon \int d^4\nu\, \bar{\psi}^{a_1^i}(\nu) \cdots \bar{\psi}^{a_k^i}(\nu) \times \psi^{b_{k+1}^i}(\nu) \cdots \psi^{b_{k+n}^i}(\nu) \times \tilde{\Phi}(x, \varepsilon, \nu) A_1^i \cdots A_k^i B_{k+1}^i \cdots B_{k+n}^i \tag{16}$$

where

$$\begin{aligned} l^i &\equiv a_1^i, a_2^i, \ldots, a_k^i, b_{k+1}^i, \ldots, b_{k+n}^i \\ D^i &\equiv A_1^i, A_2^i, \ldots, A_k^i, B_{k+1}^i, \ldots, B_{k+n}^i \end{aligned} \tag{17}$$

are the super- and subscripts of the G-functions and they constitute the quantum numbers of the ith hadron.

By the use of the Fourier transform we can define the one-particle states in the coordinate space the x space (the macrospace), and the corresponding hadron fields with their usual wave function interpretation. We define

$$\begin{aligned} |\psi(x)) &= \sum_{p_\mu=-\infty}^{\infty} \theta(-p^2)\theta(p_0)\, e^{-ipx}|\psi(p)) \\ |\psi(p)) &= \int d^4x\, e^{ipx}|\psi(x)) \end{aligned} \tag{18}$$

From (13), we can say that $\bar{\psi}^{(-)\alpha}(x)\psi^{(-)\beta}(x)G_{AB}^{ab}(x, l)$ behaves like the creation part of the quantum field of the hadron concerned, which in this case is the meson. From a similar consideration we can find the destruction part of the field to be $\bar{\psi}^{(+)\alpha}(x)\psi^{(+)\beta}(x)G_{AB}^{ab}(x, l)$. Thus, we can write the meson field as

$$\Phi_M(x) = \Phi_M^{(+)}(x) + \Phi_M^{(-)}(x) \tag{19}$$

where

$$\Phi_M^{(+)}(x) = \bar{\psi}^{(+)\alpha}(x)\psi^{(+)\beta}(x) G_{AB}^{ab}(x, l)$$
$$\Phi_M^{(-)}(x) = \bar{\psi}^{(-)\alpha}(x)\psi^{(-)\beta}(x) G_{AB}^{ab}(x, l)$$

Here M contains the quantum number of the hadron, arising from the sub- and superscripts of the ψ and G functions. The corresponding interpretation of the hadron wave function follows at once. These fields satisfy the commutation or anticommutation relations according to the spins of the hadrons to which they correspond.

3. FIELD THEORY OF HADRONS

Once the fields of the hadrons have been constituted, we can proceed to calculate the Feynman propagator functions of these fields. We calculate $\langle 0|T\Phi_\pi(x)\Phi_\pi^\dagger(y)|0\rangle$. Here,

$$\begin{aligned}\Phi_\pi(x) &= \{\bar{\psi}^{(+)\alpha}(x)\psi^{(+)\beta}(x) + \bar{\psi}^{(-)\alpha}(x)\psi^{(-)\beta}(x)\} G_{AB}^{ab}(x, l)\\ \Phi_\pi^\dagger(x) &= \{\bar{\psi}^{(+)\beta}(x)\psi^{(+)\alpha}(x) + \bar{\psi}^{(-)\beta}(x)\psi^{(-)\alpha}(x)\} G_{AB}^{\dagger ab}(x, l)\end{aligned} \tag{20}$$

$$\langle 0|T\Phi_\pi(x)\Phi_\pi^\dagger(y)|0\rangle = \langle 0|T\Phi_\pi^{(+)}\Phi_\pi^{\dagger(-)}(y)|0\rangle + \langle 0|T\Phi_\pi^{(-)}(x)\Phi_\pi^{\dagger(+)}(y)|0\rangle$$

Using (20), we obtain

$$\begin{aligned}\langle 0|T\Phi_\pi(x)\Phi_\pi^\dagger(y)|0\rangle = &-G_{AB}^{ab}(x, l) G_{AB}^{\dagger ab}(y, l)\\ &\times\{\theta(x_0-y_0) S^{(+)\beta\beta}(x-y) S^{(-)\alpha\alpha}(y-x)\\ &+\theta(y_0-x_0) S^{(+)\alpha\alpha}(y-x) S^{(-)\beta\beta}(x-y)\}\end{aligned} \tag{21}$$

where $S^{(+)}(x)$ and $S^{(-)}(x)$ are the usual anticommutator functions for the Dirac fields.

Now,

$$\begin{aligned}&-S^{(+)\beta\beta}(x-y) S^{(-)\alpha\alpha}(y-x)\theta(x_0-y_0)\\ &\quad = i^2\{(\gamma\partial_1 - m)^{\beta\beta}\Delta^{(+)}(x-y)\}\{(\gamma\partial_2 - m)^{\alpha\alpha}\Delta^{(-)}(y-x)\}\theta(x_0-y_0)\end{aligned} \tag{22}$$

where $\Delta^{(+)}(x)$ and $\Delta^{(-)}(x)$ are the commutator functions for the Klein-Gordon fields and

$$\gamma\partial_1 = \gamma^\mu \frac{\partial}{\partial x^\mu}, \qquad \gamma\partial_2 = \gamma^\mu \frac{\partial}{\partial y^\mu}$$

Now it can be proved that

$$\begin{aligned}\Delta_F(x) + i\Delta(x)\theta(-x_0) &= i\Delta^{(+)}(x)\\ \Delta_F(x) - i\Delta(x)\theta(x_0) &= -i\Delta^{(-)}(x)\end{aligned} \tag{23}$$

for the Feynman propagator $\Delta_F(x)$ of the Klein–Gordon field. Then, using (23), we have from (22),

$$S^{(+)\beta\beta}(x-y)S^{(-)\alpha\alpha}(y-x)\theta(x_0-y_0)=S_F^{\beta\beta}(x-y)S_F^{\alpha\alpha}(y-x)\theta(x_0-y_0) \quad (24)$$

Similarly,

$$S^{(+)\alpha\alpha}(y-x)S^{(-)\beta\beta}(x-y)\theta(y_0-x_0)=S_F^{\alpha\alpha}(y-x)S_F^{\beta\beta}(x-y)\theta(y_0-x_0) \quad (25)$$

where $S_F(x)$ is Feynman propagator for the Dirac field. Then substituting (24) and (25) in (21), we obtain

$$\Delta_F(x,y)\equiv\langle 0|T\Phi_\pi(x)\Phi_\pi^\dagger(y)|0\rangle=F_{AB}^{ab}(x,y,l)S_F^{\beta\beta}(x-y)S_F^{\alpha\alpha}(y-x) \quad (26)$$

with

$$F_{AB}^{ab}(x,y,l)=-G_{AB}^{ab}(x,l)G_{AB}^{\dagger ab}(y,l)$$

Thus, the propagator of the hadron (meson) is composed of the propagators arising from those of the constituents. The factor $F_{AB}^{ab}(x,y,l)$ gives rise to the extra momentum dependence and makes the propagator convergent in momentum space. In fact, in momentum space, the Fourier transform of (26) gives

$$\tilde{\Delta}_F(p)=\frac{1}{(2\pi)^8}\int \tilde{F}_{AB}^{ab}(p-p_1+p_2,l)\tilde{S}_F^{\beta\beta}(p_1)\tilde{S}_F^{\alpha\alpha}(p_2)\,d^4p_1\,d^4p_2$$

where $\tilde{\Delta}_F$, $\tilde{F}_{AB}^{ab}$, and $\tilde{S}_F$ are the Fourier transforms of the corresponding functions and it has been assumed that F_{AB}^{ab} and Δ_F are dependent on the difference of coordinates $X=x-y$ because of translational invariance. As an example, we assume

$$\tilde{F}_{AB}^{ab}(k,l)=\exp(-k^2/2l)$$

and we have

$$\Delta_F(p)=\frac{1}{(2\pi)^8}\int d^4p_1\,d^4p_2\frac{\exp\{-(1/2l)(p-p_1+p_2)^2\}}{(\gamma p_1-im)(\gamma p_2-im)} \quad (27)$$

which is convergent and gives the additional momentum dependence which ensures that the perturbation technique is applicable in hadron mechanics or in strong interactions. This was, in fact, assumed from another consideration in our earlier paper (Bandyopadhyay and De, 1975a). This situation will be more apparent in the next section, where we calculate formally the amplitude for the meson–baryon interaction. Since the hadron propagator has been found, the field theory of hadrons can be formed in the usual way, although it is necessary to find the exact form of the F-functions, that is, of the G-functions. This will be done in a future paper. Here we confine ourselves to the formal amplitude calculations of the two-body hadron processes.

4. AMPLITUDE FOR TWO-BODY HADRON REACTION AND DISCUSSION

We have already constructed the one-particle hadron state, given by (13) and (15). The two- or more-particle states can be defined as the Cartesian product of two or more one-particle states. That is, a two-particle hadron state is

$$|\psi_N(p')\psi_N(p)) = |\psi_N(p'))\otimes|\psi_N(p)) \tag{28}$$

and similarly the vacuum state is

$$|0) = |0)_{p'}\otimes|0)_p \tag{29}$$

a direct product of relevant one-particle vacuums.

Now expanding $\bar{\psi}^{(-)\alpha}(x)$ and $\psi^{(-)\beta}(x)$ in terms of the complete set of solutions, we have for the one-particle meson state, with the usual notations,

$$|\Phi_M(p)) = \int d^4x\, e^{ipx}\frac{1}{V}\sum_{\mathbf{p}_1,\mathbf{p}_2}\left(\frac{m^2}{E_{\mathbf{p}_1}E_{\mathbf{p}_2}}\right)^{1/2} e^{-i(p_1+p_2)x}$$
$$\times c^\dagger_{\mathbf{p}_2,\alpha} d^\dagger_{\mathbf{p}_1,\beta}\bar{u}_\alpha(\mathbf{p}_2)v_\beta(\mathbf{p}_1)G^{ab}_{AB}(x, l)|0)$$

If $\bar{G}^{ab}_{AB}(k, l)$ is the Fourier transform of $G^{ab}_{AB}(x, l)$, we have

$$|\Phi_M(p)) = \frac{1}{V}\sum_{\mathbf{p}_1,\mathbf{p}_2}\left(\frac{m^2}{E_{\mathbf{p}_1}E_{\mathbf{p}_2}}\right)^{1/2}\bar{u}_\alpha(\mathbf{p}_2)v_\beta(\mathbf{p}_1)$$
$$\times \bar{G}^{ab}_{AB}(p_1+p_2-p;\, l)C^\dagger_{\mathbf{p}_2,\alpha}d^\dagger_{\mathbf{p}_1,\beta}|0) \tag{30}$$

Using the usual notation

$$c^\dagger_{\mathbf{p}_2,\alpha}d^\dagger_{\mathbf{p}_1,\beta}|0) \equiv |\mathbf{p}_2, \alpha; \mathbf{p}_1, \beta)$$

we have, from (30),

$$|\Phi_M(p)) = \frac{1}{V}\sum_{\mathbf{p}_1,\mathbf{p}_2}\left(\frac{m^2}{E_{\mathbf{p}_1}E_{\mathbf{p}_2}}\right)^{1/2}\bar{u}_\alpha(\mathbf{p}_2)v_\beta(\mathbf{p}_1)$$
$$\times \bar{G}^{ab}_{AB}(p_1+p_2-p;\, l)|\mathbf{p}_2, \alpha; \mathbf{p}_1, \beta)$$
$$p_{10} = \omega_{\mathbf{p}_1}, \qquad p_{20} = \omega_{\mathbf{p}_2} \tag{31}$$

For the nucleon one-particle state we can carry out a similar procedure to obtain

$$|\psi_N(p)) = \frac{1}{V^{5/2}} \sum_{\mathbf{p}_1,\ldots,\mathbf{p}_5} \frac{m^{5/2}}{(E_{\mathbf{p}_1}\cdots E_{\mathbf{p}_5})^{1/2}}$$
$$\times v_{\alpha_1}(\mathbf{p}_1)\bar{u}_{\alpha_2}(\mathbf{p}_2)v_{\alpha_3}(\mathbf{p}_3)\bar{u}_{\alpha_4}(\mathbf{p}_4)v_{\alpha_5}(\mathbf{p}_5)$$
$$\times \bar{G}^{a_1\cdots a_5}_{A_1\cdots A_5}(p_1+\cdots+p_5-p, l)|\mathbf{p}_1, \alpha_1; \ldots; \mathbf{p}_5, \alpha_5) \quad (32)$$

In the case of massless constituents the normalization is to be changed in accord with the corresponding changes for the spinors. Also,

$$\bar{G}^{a_1\cdots a_5}_{A_1\cdots A_5}(k, l) \qquad \text{and} \qquad \bar{G}^{ab}_{AB}(k, l)$$

are actually functions of $(\mathbf{k}, l)$ because $K_0 = p_{10}+p_{20}+\cdots+p_{50}-p_0$, with $p_{i0} = (\mathbf{p}_i^2+m^2)^{1/2}$ and $\mathbf{k} = \mathbf{p}_1+\mathbf{p}_2+\cdots+\mathbf{p}_5-\mathbf{p}$.

Now we insist on the following expansion:

$$\bar{G}^{a_1\cdots a_5}_{A_1\cdots A_5}(p_1+\cdots+p_5-p, l)$$
$$= \sum_{\substack{\text{Perm. of}(1,2),\\ (3,4)\,\&\,5}} \sum_{p',p'',p'''} \bar{G}^{a_1a_2}_{A_1A_2}(k_1-p', l)$$
$$\times \bar{G}^{a_3,a_4}_{A_3A_4}(k_2-p'', l)\bar{G}^{a_5}_{A_5}(p_5-p''', l)\bar{G}(p'+p''+p'''-p, l) \quad (33)$$

with

$$p_1+p_2 = k_1, \qquad p_3+p_4 = k_2 \quad (33a)$$

In coordinate space this expansion is equivalent to the following decomposition:

$$G^{a_1\cdots a_5}_{A_1\cdots A_5}(x, l) = \sum_{\substack{\text{perm. of}(1,2),\\ (3,4)\,\&\,5}} (2\pi)^{12} G^{a_1a_2}_{A_1A_2}(x, l)G^{a_3a_4}_{A_3A_4}(x, l)G^{a_5}_{A_5}(x, l) \quad (34)$$

Thus we can write the one-particle nucleon state as, using (33) and (34),

$$|\psi_N(p))$$
$$= \frac{1}{V^{5/2}} \sum_{\mathbf{p}_1,\ldots,\mathbf{p}_5} \frac{m^{5/2}}{(E_{\mathbf{p}_1}\cdots E_{\mathbf{p}_5})^{1/2}}$$
$$\times v_{\alpha_1}(\mathbf{p}_1)\bar{u}_{\alpha_2}(\mathbf{p}_2)v_{\alpha_3}(\mathbf{p}_3)\bar{u}_{\alpha_4}(\mathbf{p}_4)v_{\alpha_5}(\mathbf{p}_5)$$
$$\times \sum_{\substack{\text{Perm. of}(1,2),\\ (3,4)\,\&\,5}} \sum_{p',p'',p'''} \bar{G}^{a_1a_2}_{A_1A_2}(k_1-p', l)$$
$$\times \bar{G}^{a_3a_4}_{A_3A_4}(k_2-p'', l)\bar{G}^{a_5}_{A_5}(p_5-p''', l)\bar{G}(p'+p''+p'''-p, l)$$
$$\times |\mathbf{p}_1, \alpha_1; \mathbf{p}_2, \alpha_2)\otimes|\mathbf{p}_3, \alpha_3, \mathbf{p}_4, \alpha_4)\otimes|\mathbf{p}_5, \alpha_5)$$
$$= \sum_{\text{perm.}} \sum_{p',p'',p'''} \bar{G}(p'+p''+p'''-p, l)|\Phi_{M_1}(p')\Phi_{M_2}(p'')\psi_f(p''')) \quad (35)$$

where $|\psi_f(p''')\rangle$ represents the one-particle fermion state and $\sum_{\text{perm.}}$ is the sum over the meson states M_1 and M_2 whose constituents are rearranged from the five lepton constituents of the nucleon.

Now let us sketch the two-body meson–baryon reaction (MB→MB), for example, $\pi N \to N\rho$, πN, etc. The initial and final two-body states are the direct products of the two one-particle states as described earlier. Thus, the two-particle meason-nucleon state is

$$|\psi_N(p_1)\rangle \otimes |\Phi_M(p_2) = |\psi_N(p_1)\Phi_M(p_2)\rangle$$

and the reaction amplitude for the two-body meson–baryon interaction is given by

$$(\psi_N(p_1')\Phi_M(p_2')|S|\psi_N(p_1)\Phi_M(p_2)) \equiv A(p_1', p_2', p_1, p_2) \tag{36}$$

with $S = U(\infty, -\infty)$, where U is the usual time-displacement operator. Using (35), we have

$$\begin{aligned} A(p_1', p_2', p_1, p_2) = \sum_{\text{perm.}} \sum_{p_f', p_f'', p_f'''} \sum_{p_i', p_i'', p_i'''} & \bar{G}^{\dagger}(p_f' + p_f'' + p_f''' - p_1', l) \\ & \times \bar{G}(p_i' + p_i'' + p_i''' - p_1; l) \\ & \times (\Phi_M(p_f')\Phi_M(p_f'')\psi_f(p_f''')\Phi_M(p_2')|S|\Phi_M(p_i') \\ & \times \Phi_M(p_i'')\psi_f(p_i''')\Phi_M(p_2)) \end{aligned} \tag{37}$$

Now $S = U(\infty, -\infty)$, where $U(t, t_0)|a, t_0\rangle = |a, t\rangle$, with $|a, t_0\rangle$ and $|a, t\rangle$ being the physical states at times t_0 and t, respectively. Now, as the state $|a, t\rangle$ is the state in the product space, we write

$$\begin{aligned} |a, t_0\rangle &= |a', t_0\rangle \otimes |a'', t_0\rangle \\ |a, t\rangle &= |a', t\rangle \otimes |a'', t\rangle \end{aligned} \tag{38}$$

Then

$$|a, t\rangle = U(t, t_0)|a', t_0\rangle \otimes |a'', t_0\rangle \tag{39}$$

Let U' and U'' be the time displacement operators that operate on the states $|a', t\rangle$ and $|a'', t\rangle$, respectively. Then,

$$\begin{aligned} |a, t\rangle = |a', t\rangle \otimes |a'', t\rangle &= U'(t, t_0)|a', t_0\rangle \otimes U''(t, t_0)|a'', t_0\rangle \\ &= U'(t, t_0)\, U''(t, t_0)|a', t_0\rangle \otimes |a'', t_0\rangle \end{aligned} \tag{40}$$

From (39) and (40), we have

$$U(t, t_0) = U'(t, t_0)\, U''(t, t_0) \tag{41}$$

and hence

$$U(\infty, -\infty) = U'(\infty, -\infty)\, U''(\infty, -\infty) \tag{41a}$$

or

$$
\begin{aligned}
&S = \tilde{S} \cdot S' \\
&\tilde{S} = U'(\infty, -\infty), \qquad S' = U''(\infty, -\infty)
\end{aligned}
\tag{42}
$$

Then the amplitude can be written as

$$
\begin{aligned}
&A(p_1', p_2', p_1, p_2) \\
&\quad = \sum_{\text{perm.}} \sum_{p_f', p_f'', p_f'''} \sum_{p_i', p_i'', p_i'''} \bar{G}^\dagger(p_f' + p_f'' + p_f''' - p_1', l) \\
&\quad \times \bar{G}(p_i' + p_i'' + p_i''' - p_1, l) \\
&\quad \times \{(\Phi_M(p_f')\Phi_M(p_2')|\tilde{S}|\Phi_M(p_i')\Phi_M(p_2))\} \\
&\quad \times \{(\Phi_M(p_f'')\psi_f(p_f''')|S'|\Phi_M(p_i'')\psi_f(p_i'''))\}
\end{aligned}
\tag{43}
$$

Here $\tilde{S}$ corresponds to the S-operator, which is responsible for the $\pi\pi$ interaction (that is, $\pi\pi \to \rho \to \pi\pi$, $\pi\omega$, etc.) and S' corresponds to only the "connector part", which is the rearrangement of the "spectator" (Bandyopadhyay and De, 1975a) to the reaction. Note that the meson–baryon interaction is thus governed by the $\pi\pi$ interaction, that is, by the operator part $\tilde{S}$ only. In our earlier papers (Bandyopadhyay and De, 1973, 1975a, b; De, 1983) it was conjectured that the contribution from the spectator part of the form:

$$
(|S'|) = \delta(p_f'' - p_i'')\delta(p_f''' - p_i''')
$$

In fact, in that formalism, the amplitude is

$$
\begin{aligned}
A(p_1', p_2', p_1, p_2) = \sum_{\text{perm.}} \sum_{p_f', p_f'', p_f''', p_i'} & \bar{G}^\dagger(p_f' + p_f'' + p_f''' - p_1', l) \\
& \times \bar{G}(p_i' + p_f'' + p_f''' - p_1, l) K(p_f', p_2', p_i', p_2)
\end{aligned}
\tag{44}
$$

where

$$
K(p_f', p_2', p_i', p_2) = (\Phi_M(p_f')\Phi_M(p_2')|\tilde{S}|\Phi_M(p_i')\Phi_M(p_2)) \tag{45}
$$

Apart from the dependence on the amplitude K of the $\pi\pi$ interaction, the amplitude of the meson–baryon reaction depends on the factor $\bar{G}^\dagger \cdot \bar{G}$, which provides additional momentum dependence. This momentum dependence can be compared with the form factor as well as the rearrangement amplitude $T(s, t)$, an amplitude related to the rearrangement of partons (constituents) involved in duality diagrams that was conjectured in our previous model (Bandyopadhyay and De, 1973, 1975a, b; De, 1983).

In fact, the term $T(s, t)$ behaves like $s^{-n\gamma}$ for large s, where n is the number of constituents of hadrons that are rearranged and γ is a parameter that has a correspondence with the Regge amplitude. Thus, the effective coupling term in the amplitude becomes $g(s) = g \cdot s^{-n\gamma}$, and inserting this factor for each vertex in the perturbative expansion, it can be seen that the final expression can be made convergent because the higher order terms will not be large enough to contribute. Consequently, we do not face any inconsistency as in the naive form of the field theory. Also, the form factor $F(t)$ arises because we have assumed that the basic unit of the strong interaction is the π-meson, which in itself is not a fundamental particle but is composed of a muon-antimuon pair, and it has been shown earlier (De, 1977) that physically it corresponds to the electromagnetic form factor of the pion. Thus, the amplitude is of the form

$$A(MB \to MB) = \alpha A(\pi\pi \to \pi\pi(\omega)) T(s, t) \tag{46}$$

where α is a numerical factor depending on the number of interacting pions in the structure of M and B. Note that this model is in good agreement (Bandyopadhyay and De, 1973, 1975a, b) with the experimental results. Now if we compare this amplitude with that of the present model, that is, with (44), we find that $\bar{G}^{\dagger}\bar{G}$ contributes to the rearrangement term $T(s, t)$ a factor that was included in an ad hoc manner in our previous model. Thus, the present model provides a foundation for this rearrangment term. Of course, it remains a problem to ascertain the form of the function G, which essentially depends on the nature of the field in the microdomain and on the "inner" amplitude function in forming the particle space. Such considerations will be discussed in a subsequent paper.

ACKNOWLEDGMENTS

The author wishes to thank Prof. K. P. Sinha for his encouragement, invitation, and hospitality at the Centre of Theoretical Studies, Indian Institute of Science, Bangalore, where the final part of this work was completed.

REFERENCES

Bandyopadhyay, P. (1984). *Hadronic Journal*, **7**, 1706.
Bandyopadhyay, P., and De, S. S. (1973). *Nuovo Cimento*, **14A**, 285.
Bandyopadhyay, P., and De, S. S. (1975a). *Nuovo Cimento*, **27A**, 294.
Bandyopadhyay, P., and De, S. S. (1975b). *Acta Physica Polonica*, **B6**, 241.
Chowdhuri, V. S. P. (1981). *Bulletin of Calcutta Mathematical Society*, **73**, 11.
De, S. S. (1977). *Acta Physica Polonica*, **B8**, 521.
De, S. S. (1983). Preprint, Calcutta University.
De, S. S. (1985). *Indian Journal of Pure and Applied Mathematics*, **16**(8), 909.
Rund, H. (1959). *The Differential Geometry of Finsler Spaces*, Springer-Verlag, Berlin.